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Quantum Fluctuations and Variable G Return Einstein's Field Equation to Its Original Formulation

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Abstract The standard ΛCDM model has successfully depicted most of the astronomical observations. However, the model faces several question marks such as, what was the cause of the Big Bang singularity, what is the physics behind dark matter? The origin of dark energy is still unclear. The present theory, CBU, standing for the Continuously Breeding Universe, has been developed along with known principles of physics. The theory incorporates important ideas from the past. The universe is a complex emerging system, which starts from the single fluctuation of a positron-electron pair. Expansion is driven by the emersion of new pairs. The gravitational parameter G is inversely proportional to the Einsteinian curvature radius r. The Planck length ℓ_P and Planck time t_P are dependent of the curvature and accordingly by the size of the universe. It is shown that the solution to the Schrödinger equation of the initial positron-electron fluctuation includes an exponential function parameter equal to the Planck length of the initial event. The existence of a wave function provides a link between quantum mechanics and the theory of general relativity. The fast change of momentum increases the Heisenberg uncertainty window thereby enhancing the positron-electron pair production, especially strong in the early universe. When these findings are introduced in the energy-momentum tensor of Einstein's Field Equation, the equation acquires a simple configuration without G and a cosmological constant. The universe is a macroscopic manifestation of the quantum world.

Keywords: *general relativity, quantum fluctuations, Planck scale, variable gravitational constant, Robertson-Walker metrics*

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1. Introduction

The renowned British physicist Paul A. M. Dirac was interviewed in an early 1970's CBC radio series entitled Physics and Beyond. Referring to his work on astrophysics, Dirac said: "Now, I want to have G varying, and I also want to have a continuous creation. It's possible to combine those two ideas and I've worked out some equations on possible models of the universe incorporating them," [1]. These ideas remained on a general level in an article published in 1974, [2]. A year earlier E. P. Tryon had suggested that the universe was initiated by a positron-electron quantum fluctuation, [3].

In 1994 Alan Guth, the man behind the inflation theory, suggested that the total energy of the universe is zero. Matter and radiation provide the positive part, while the potential energy of gravity forms the negative part, [4].

The present article exposes a comprehensive study of a scenario based on these presumptions. Quantum mechanics will have a key position in the explanation of the universe expansion. The gravitational force is a macroworld phenomenon, a force created by a disturbance in the gradient of a field, in this case the curvature of space. Similarly, macroscopic electro-magnetic forces obey Maxwell's laws and not the flow of photons.

The energy-momentum tensor of the Einstein Field Equation will be formulated in the original form $G_{\mu\nu} = T_{\mu\nu}$, [5]. The momentum change, expansion pressure and G (gravitational parameter) must be included in the T-tensor density and pressure terms in order to find the correct formulation.

The basis of the present article was outlined in four previous papers, [6,7,8,9]. The *CBU- theory (Continuously Breeding Universe)* was developed in the first one along with a Coriolis-explanation of galaxy dynamics. In the following papers momentum change was suggested as an alternative to the cosmological constant, the redshift time scaling was revised due to the larger G in past times. A solution to the Schrödinger equation of the initial event was shown to have a crucial impact on the excitation of real positron-electron pairs from the virtual ground state foam.

According to the CBU theory the energy is confined to matter and radiation, neither dark energy nor dark matter is required. However, a certain part of the quantum foam, from which real matter emerges, may be considered as virtual dark energy, which in an accumulated form corresponds to the dark energy of the ΛCDM theory.

2. Fundamental Equations

2.1. The Postulate

In principle the universe can be considered as a black hole, because light is confined to the space of the universe, there is no space on the outside. Bernard McBryan has studied black holes of different modes, [10]. He states that one could live in a low-density black hole without knowing it. According to his classification our universe could be a "classical finite height black hole", wherein the photon sphere radius is half the Schwarzschild radius r_s. We make this a fundamental law by utilizing the Schwarzschild photon horizon equation

$$r_u = \frac{1}{2} r_s = \frac{GM_u}{c^2}, \tag{1}$$

where r_u is the outer radius and M_u the mass of the universe (actually the total energy W_u divided by c^2), G is the gravitational parameter and c the velocity of light. Equation (1) has occurred in several significant proposals, most important those by Brans and Dicke, [11], Sciama, [12], and Dirac, [2]. A simple formula also hints in the same direction: Imagine all mass concentrated to the centre, how far do we have to go in order to create a new mass m, which equals the gravitational potential energy? Answer: $mc^2 = GM_u m/r_u$, i.e. eq. (1).

However, the best argument in favour of eq. (1) is the Schwarzschild photon sphere radius.

It has always been a problem to visualize the 4D space-time into 3D. In the paper on the cosmological constant of 1917, [13], Einstein writes "the points of this hyper-surface form a three-dimensional continuum, a spherical space of curvature R". He also defines a constant $\kappa = 8\pi G/c^2$, which scales the energy-momentum tensor $T_{\mu\nu}$. As will be shown later this constant is directly connected to a 3D sphere. Figure 1. is a visualization of the 3D universe, the radius r_u is the virtual outer radius, even if there is no outer border as the space turns into itself. Outside the universe there is no space, no vacuum.

However, it is useful to think of the universe as a sphere, the volume and outer area of which are

$$V_u = \frac{32\pi r^3}{3}, \tag{2}$$

$$A_u = 16\pi r^2. \tag{3}$$

Here $r = r_u/2$ is the radius of the observable universe. $r = ar_0$ is the most important variable of this study. a is the scale factor, r_0 the present r.

Based on eq. (1) we state as our **principal postulate** the following equation

$$\frac{G}{r} = \frac{2c^4}{W_u}, \tag{4}$$

where $W_u = M_u c^2$, the total real energy (matter and radiation) of the universe.

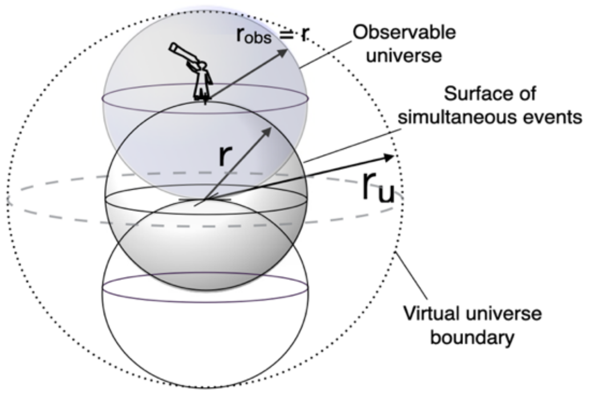

Figure 1. Definitions related to the spherical 3D image of the universe

2.2. Consequences

At the initial moment, when the first positron-electron pair appears, we have

$$\frac{G_i}{r_i} = \frac{2c^2}{2m_e} = \frac{c^2}{m_e}, \tag{5}$$

where G_i is the initial gravitational parameter, r_i the radius and m_e (W_e/c^2) the electron rest mass. The energy equation is obtained by assuming a πr_i separation between the particles

$$2m_e c^2 = G_i \frac{m_e^2}{\pi r_i} + \frac{e^2}{4\pi\varepsilon_0 (\pi r_i)}, \tag{6}$$

here e is the electron charge and ε_0 is the vacuum permittivity. The (curvature, see section 7.3.) radius of the initial universe is

$$r_i = \frac{e^2}{4\pi\varepsilon_0 m_e c^2 (2\pi - 1)} = 0{,}53337904 \cdot 10^{-15} m, \tag{7}$$

As we shall see later, r_i is a fundamental quantity in the physics of the universe.

The following relation can be confirmed, cf. [9],

$$G_i r_i = \frac{r_i^2 c^4}{W_e} = constant. \tag{8}$$

As a result, the present value of the radius of the observable universe can be determined

$$r_0 = \frac{1}{G_0} \cdot \frac{r_i^2 c^4}{W_e} = 4{,}2055083 \cdot 10^{26} m, \tag{9}$$

where $G_0 = 6{,}6743015 \cdot 10^{-11}$ Nm^2/kg^2, Newton's gravitational constant. r_0 is close to the official estimate of $4{,}40 \cdot 10^{26}$ m.

When G_0 from eq. (9) is substituted into eq. (4) we obtain the energy of the universe

$$W_{u0} = 2W_e \left(\frac{r_0}{r_i}\right)^2 = 1{,}017948 \cdot 10^{71} J \sim 1{,}132620 \cdot 10^{54} kg. \tag{10}$$

These numbers are consistent with the official baryonic content (here 8 times those of the observable universe).

Eq. (10) is valid for any arbitrary value of r, i.e. the energy is proportional to r^2. The author has chosen the following definition

$$W_u = 4\pi b r^2, \tag{11}$$

where b is a universal energy "pressure" constant (J/m^2). From the initial event we have

$$b = \frac{2W_e}{4\pi r_i^2} = 0,45801387 \cdot 10^{17}\ J/m^2. \tag{12}$$

In summary we write down some relations that are of importance in the rest of the study. The gravitational parameter is

$$G(r) = \frac{c^4}{2\pi b r}. \tag{13}$$

The density of matter (m) and radiation (γ) is

$$\rho_{m\gamma} = \frac{W_u}{V_u c^2} = \frac{3b}{8rc^2}. \tag{14}$$

In the original General Relativity text, [5], Einstein introduced what he called the Eulerian hydrodynamic pressure P_E, which is obtained from $dW_u = -P_E dV_u$. We have

$$P_E = -\frac{b}{4r}. \tag{15}$$

The minus-sign indicates that the pressure drives expansion (capital letter P for pressure while p symbolizes momentum).

3. The Hubble Parameter

The Hubble parameter h_H (1/s) reflects the velocity of expansion. For a constant rh_H the time derivative is zero. It was shown, cf. Perlmutter, [14], that this is not the case. Starting from the Friedmann-Robertson-Walker kinetic energy equation a rigorous derivation of the acceleration is presented in ref. [7]. Here we use the result as follows:

Ansatz

$$g = r_0 \ddot{a} = \frac{c^2}{2r_0 a} B, \tag{16}$$

where B equals (ß – 1) in ref. [7]. In section 6 it is shown that B is not an arbitrarily chosen constant, but a calculable factor connecting expansion to the positron-electron inflow.

The acceleration symbol g was chosen to avoid a mix-up with the scale factor *a*. The acceleration parameter B is approximately a constant, a fraction of 1.

Equation (16) is differentiated with respect to $\dot{a}$

$$\dot{a} d\dot{a} = \frac{c^2}{r_0^2} B \frac{da}{a}. \tag{17}$$

By integration we obtain

$$\dot{a} = \frac{c}{r_0}\sqrt{B \ln \frac{a}{a_i}} = \frac{c}{r_0}\sqrt{B \ln \frac{r}{r_i}}, \tag{18}$$

where $a_i = r_i/r_0$.

The incoming new matter causes a continuous change in the momentum p. As a result, there is an additional pressure P_M enhancing the expansion. The momentum force is

$$\begin{aligned} F_M &= \frac{dp}{dt} = M_u \frac{\partial v}{\partial t} + v \frac{\partial M_u}{\partial t} \\ &= M_u \frac{\partial v}{\partial r}\frac{dr}{dt} + v \frac{\partial M_u}{\partial r}\frac{dr}{dt} = 8\pi b r B\left(\ln \frac{r}{r_i} + \frac{1}{4}\right). \end{aligned} \tag{19}$$

We incorporate ¼ into eq. (18) and write $L = \ln(r/r_i) + ¼$. This does not change the acceleration, because integration excludes the constant.

The Hubble parameter is now obtained from

$$h_H = \frac{\dot{a}}{a} = \frac{c}{r}\sqrt{B\left(\ln\frac{r}{r_i} + \frac{1}{4}\right)} = \frac{c}{r}\sqrt{BL}. \tag{20}$$

The pressure provided by the increasing momentum is

$$P_M = \frac{F_M}{A_u} = -\frac{bBL}{2r}. \tag{21}$$

The acceleration parameter B is still unknown, the determination requires a deeper investigation of the quantum mechanics of the positron-electron generation.

4. The Schrödinger Equation

This section offers a bridge between quantum mechanics and gravity and is crucial for the understanding of the emergence of the universe.

The time-independent Schrödinger equation is

$$\nabla^2 \psi + \frac{8\pi^2 m_e}{h^2}(E_i - U_i)\psi = 0, \tag{22}$$

where ψ is the quantum-mechanical wave function, E_i is the ground state energy, $U_i = 4\pi b r_i^2$ is the potential energy. The spherically symmetric form of eq. (22) is

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r}\frac{\partial \psi}{\partial r} + \frac{8\pi^2 m_e}{h^2} E_i \psi - \frac{32\pi^3 m_e b}{h^2} r^2 \psi = 0, \tag{23}$$

where r is the curvature radius and b the energy constant of eq. (12).

Let

$$\frac{a_1^4}{4} = \frac{h^2}{32\pi^3 m_e b}, \tag{24}$$

$$C = \frac{8\pi^2 m_e}{h^2} E_i. \tag{25}$$

The Schrödinger equation takes the form

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r}\frac{\partial \psi}{\partial r} + C\psi - \frac{4}{a_1^4} r^2 \psi = 0. \tag{26}$$

This Sturm-Liouville type differential equation has the solution

$$\psi(r)=\frac{1}{r}e^{-\left(\frac{r}{a_1}\right)^2}\left[c_1 H_n\left(\frac{r}{a_1}\right)\right] + \frac{1}{r}e^{-\left(\frac{r}{a_1}\right)^2}\left\{c_2 1F1_{(a;b;x)}\left[\frac{1}{4}\left(1-Ca_1^2\right);\frac{1}{2};\left(\frac{r}{a_1}\right)^2\right]\right\}, \quad (27)$$

where H_n is the Hermite polynomial function and $1F1_{(a;b;x)}$ is the Kummer confluent hypergeometric function. c_1 and c_2 are constants of integration.

We have a special interest in the constant a_1

$$a_1=\sqrt{\frac{h}{2\pi}\frac{1}{\sqrt{2\pi b m_e}}}. \quad (28)$$

By substituting b from eq. (12) we have

$$\frac{1}{\sqrt{2\pi b m_e}}=\frac{r_i}{m_e c}. \quad (29)$$

And by substituting G_i from eq. (5) we obtain

$$a_1=\sqrt{\frac{hG_i}{2\pi c^3}}=\ell_{Pi}. \quad (30)$$

By definition ℓ_{Pi} is the Planck length of the virgin universe. In the CBU theory the Planck length is dependent on the radius r – and time. The numerical value is $\boldsymbol{\ell_{Pi}}$ **= 1,435164·10^{-14} m**, that is 26,907 times the initial radius r_i.

This is an important result, which proves that the postulate and related presumptions of the CBU theory are credible.

In order to show the link between the initiation of the universe and the quantum fluctuation we establish the ratio

$$\frac{\ell_{Pi}^2}{r_i r_B}=\frac{hr_i}{2\pi m_e c}\cdot\frac{1}{r_i}\cdot\frac{\pi m_e e^2}{\varepsilon_0 h^2}=\frac{e^2}{2\varepsilon_0 hc}, \quad (31)$$

where r_B is the Bohr radius of the hydrogen atom. The last term equals the fine structure constant α_{fs}, we have

$$\alpha_{fs}=\frac{\ell_{Pi}^2}{r_i r_B}. \quad (32)$$

The equation is an evidence of the connection between quantum mechanics and gravity. It emphasizes the significance of the curvature radius r_i of the virgin universe.

Substituting G from eq (13) into eq. (30) we obtain a general expression for the Planck length

$$\ell_P(r)=\ell_{\mathrm{Pr}}=\frac{1}{2\pi}\sqrt{\frac{hc}{br}}. \quad (33)$$

As a control the present value of the Planck length as calculated according to eq. (33) using r_0 = 4,205508·10^{26} m results in 1,616255·10^{-35}, which is the official value as expected.

In analogy with the hydrogen atom the ground state energy E_i is postulated to be of the form

$$E_i=\frac{h^2}{8m_e\ell_{Pi}^2}=\frac{hc}{4\pi r_i}. \quad (34)$$

The general expression of the instantaneous ground state energy takes the form

$$E_{GS}(r)=E_{GSr}=\frac{h^2}{8m_e\ell_{Pi}\ell_{Pr}}=\frac{\pi hc\ell_{Pi}}{4\ell_{\mathrm{Pr}}r_i}=\pi^2\frac{\ell_{Pi}}{2r_i}\sqrt{hcbr} \quad (35)$$

It is plausible to assume that E_{GSr} represents the instantaneous value of the quantum foam, the existence of which was first suggested by John Archibald Wheeler, [15], i.e. the virtual vacuum energy.

The Planck energy W_P has an important bearing on the inflow of new matter. By definition

$$W_{Pi}=\sqrt{\frac{hc^5}{2\pi G_i}}=\frac{\ell_{Pi}}{r_i}W_e. \quad (36)$$

At the initial moment a positron and an electron are exited, from eq. (36) one deduces that the particles originate from

$$2W_e=\frac{W_{Pi}}{\ell_{Pi}/2r_i}. \quad (37)$$

The ratio $\boldsymbol{\ell_{Pi}/2r_i}$ **= 13,453499** has a significant role in the CBU theory, cf. section 6.

The general expression for $W_P(r) = W_{\mathrm{Pr}}$ is

$$W_{\mathrm{Pr}}=\sqrt{hcbr}. \quad (38)$$

The present value is 1,9560815·10-9 J, which is in full compliance with the official value.

A comparison of eq. (38) with eq. (35) shows that

$$E_{GSr}=\pi^2\frac{\ell_{Pi}}{2r_i}W_{Pr}. \quad (39)$$

W_{Pr} is a constant fraction of the ground state energy.

5. Generalized Uncertainty

A generalized form of the Heisenberg uncertainty principle has an important impact on the production of new e^+-e^- pairs. Because of the dynamic change of the momentum the uncertainty window is much wider than that provided by the classical Heisenberg formulation, cf. Adler [16]. We divide Δx into a Heisenberg component $\Delta x_H = h/(4\pi\Delta p)$ and a gravity component

$$\Delta x_g=\frac{h}{4\pi\Delta p_g}. \quad (40)$$

The momentum uncertainty is

$$\Delta p=\frac{dp}{dr}\ell_{Pr}. \quad (41)$$

We have

$$\frac{dp}{dr} = \frac{8\pi br}{c}\sqrt{BL}\left(1+\frac{1}{4L}\right), \tag{42}$$

where the term containing 1/4L is due to the fact that L is a ln(r) function.

Assuming that the location uncertainty is $\Delta x = \ell_{Pr}/2$ we have

$$\Delta p \Delta x_H \leq \frac{1}{2}\frac{dp}{dr}\ell_{Pr}{}^2 = \frac{h}{4\pi} f_r, \tag{43}$$

where

$$f_r = 4\sqrt{BL}\left(1+\frac{1}{4L}\right). \tag{44}$$

Ronald J. Adler has derived an expression for the gravity component, [16],

$$\Delta x_g = 2\pi\frac{\Delta p \ell_{Pr}^2}{h} = \frac{h}{4\pi}\left[\frac{2}{\Delta p}\left(\frac{2\pi\ell_{Pr}\Delta p}{h}\right)^2\right] = \frac{h}{4\pi\Delta p_g}. \tag{45}$$

After some algebraic manipulation we arrive at the final uncertainty equation

$$\Delta p \Delta x = \Delta p\left(\Delta x_H + \Delta x_g\right) \leq \frac{h}{4\pi}\left(f_r + f_r^2\right) = \frac{h}{4\pi}F_r, \tag{46}$$

where $F_r = f_r(1 + f_r)$ is the overall uncertainty factor.

6. Virtual and Real Energy

The pressure responsible for the expansion can be formulated as a negative energy density. By combining eqs (14), (15) and (21) we obtain

$$\begin{aligned} -\frac{P_{EM}}{c^2} &= -\frac{P_E}{c^2} - \frac{P_M}{c^2} = \frac{b}{4c^2 r}(1+2BL) \\ &= \frac{2}{3}\rho_{m\gamma}(1+2BL) = \rho_{vDE}. \end{aligned} \tag{47}$$

The energy $W_{vDE} = \rho_{vDE} V_u c^2$ is the virtual dark energy. It is an accumulation of the energy fraction of E_{GSr}, from which the real energy W_u emerges. As will be shown below, the ratio of the densities equals $\ell_{Pi}/2r_i$,

$$\frac{W_{vDE}}{W_u} = \frac{\rho_{vDE}}{\rho_{m\gamma}} = \frac{\ell_{Pi}}{2r_i} = \frac{2}{3}(1+2BL). \tag{48}$$

We can now determine the present value of **BL = 9,59012425** and **B = 0,099152544** ($L=\ln(r_0/r_i)+1/4 =$ 96,72091). According to eq. (48) BL is a constant, which underlines the fact that the acceleration Ansatz is an approximation.

From eq. (20) we obtain the Hubble parameter

$$h_H = \frac{c}{r_0}\sqrt{BL} = 2,2076\cdot 10^{-18}s^{-1}$$

$$\sim H_0 = 68,118\frac{km}{sMpc}.$$

In order to derive the real energy change dW_u/dt we postulate that the change is proportional to the Planck energy divided by the Planck time ($t_{Pr} = \ell_{Pr}/c$) times the uncertainty "enhancement" F_r. We have

$$\begin{aligned} \frac{dW_u}{dt} &= \frac{1}{\ell_{Pi}/2r_i}\frac{dW_{EM}}{dt} = \frac{1}{\ell_{Pi}/2r_i}\frac{cW_{Pr}}{\ell_{Pr}}F_r \\ &= \frac{1}{\left(\ell_{Pi}/2r_i\right)^2}\frac{cE_{GSr}}{\pi^2\ell_{Pr}}F_r. \end{aligned} \tag{49}$$

We substitute E_{GSr} from eq. (35) (2nd middle term) and F_r from eqs (44) and (46), and end up with

$$\frac{dW_u}{dt} = 8\pi brc\sqrt{BL}\,\frac{(1+1/4L)\left[1+4\sqrt{BL}(1+1/4L)\right]}{\ell_{Pi}/2r_i}. \tag{50}$$

The numerator appears to be $(1+1/4L)[1+4\sqrt{BL}(1+1/4L)]=$ 13,45387, that is an accuracy of 5 digits as compared to $\ell_{Pi}/2r_i$ = 13,453499. The result proves that dW_u/dt equals the time derivative of $W_u = 4\pi br^2$, i.e.

$$\frac{dW_u}{dt} = 8\pi brc\sqrt{BL}. \tag{51}$$

This important result shows that the energy input can be deduced directly from a quantum mechanical origin.

7. General Relativity

7.1. The Einstein Field Equation

The Field Equation in its simplest form is

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{52}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor. Eq. (52) has an analytical solution, if $T_{\mu\nu}$ is isotropic and homogeneous.

In the present study we expel the cosmological constant Λ term. It will be shown that incoming new matter in combination with a variable G influences the energy density and the pressure responsible for the expansion.

The Einstein tensor is divided into the Ricci curvature tensor $R_{\mu\nu}$ and the Ricci scalar R. We have

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}. \tag{53}$$

The components of the tensor are obtained by utilizing the Christoffel symbols, the procedure can be found in any textbook on General Relativity, cf. Carroll [17]. The Ricci tensor is

$$R_{\mu\nu}=\begin{pmatrix}-3\frac{\ddot{a}}{a} & 0 & 0 & 0\\ 0 & \left(\frac{\ddot{a}}{a}+2\left(\frac{\dot{a}}{a}\right)^2+2k\frac{c^2}{a^2r_{cur}^2}\right) & 0 & 0\\ 0 & 0 & \left(\frac{\ddot{a}}{a}+2\left(\frac{\dot{a}}{a}\right)^2+2k\frac{c^2}{a^2r_{cur}^2}\right) & 0\\ 0 & 0 & 0 & \left(\frac{\ddot{a}}{a}+2\left(\frac{\dot{a}}{a}\right)^2+2k\frac{c^2}{a^2r_{cur}^2}\right)\end{pmatrix}, \quad (54)$$

where r_{cur} is the curvature radius and k the curvature parameter according to the Friedmann-Robertson-Walker metrics. k is 1 for a closed, 0 for a flat and -1 for an open universe.

The Ricci scalar is

$$R=-6\left[\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2+k\frac{c^2}{a^2r_{cur}^2}\right]. \quad (55)$$

The metric tensor $g_{\mu\nu}$ is

$$g_{\mu\nu}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}. \quad (56)$$

7.2. The Energy-momentum Tensor

Judging from eq. (52) and (55) the unit of the energy-momentum tensor $T_{\mu\nu}$ components is that of a mass density, kg/m^3. The tensor components are zero except for the diagonal. We define the tensor in the following way

$$T_{\mu\nu}=\begin{pmatrix}\rho_{eq} & 0 & 0 & 0\\ 0 & \frac{P_{\exp}}{c^2} & 0 & 0\\ 0 & 0 & \frac{P_{\exp}}{c^2} & 0\\ 0 & 0 & 0 & \frac{P_{\exp}}{c^2}\end{pmatrix}. \quad (57)$$

Our task is to find the expressions for the equivalent density ρ_{eq} and the pressure P_{exp} responsible for both expansion and acceleration.

The density ρ_{eq} has a bearing on the velocity of expansion, i.e. the Hubble parameter. It consists of two components

$$\rho_{eq}=\rho_{m\gamma}+\rho_{EM}, \quad (58)$$

where ρ_{EM} is caused by the momentum change.

The 1st law of thermodynamics says that

$$\frac{dW_{\exp}}{dt}+P_{EM}\frac{dV}{dt}=0, \quad (59)$$

where $W_{exp} = \rho_{EM}c^2V$. The time derivative is

$$\frac{dW_{\exp}}{dt}=\dot{\rho}_{EM}c^2V+\rho_{EM}c^2\frac{dV}{dt}, \quad (60)$$

here $dV/dt = 3V\frac{\dot{a}}{a}$. Further we approximate that

$$\dot{\rho}_{EM}\cong-\rho_{EM}\frac{\dot{a}}{a}. \quad (61)$$

From eq. (59) we now obtain

$$\rho_{EM}=-\frac{3}{2}\frac{P_{EM}}{c^2}. \quad (62)$$

Substituting P_{EM} from eq. (47) we have

$$\rho_{EM}=\rho_{m\gamma}\left(1+2BL\right), \quad (63)$$

and

$$\rho_{eq}=\rho_{m\gamma}+\rho_{EM}=2\rho_{m\gamma}\left(1+BL\right). \quad (64)$$

In order to get the total pressure P_{exp} we need to include the gravitational parameter G into the 1st law of thermodynamics

$$\frac{d\left(GW_{\exp}\right)}{dt}+GP_{\exp}\frac{dV}{dt}=0. \quad (65)$$

After some manipulations we have

$$\frac{\dot{G}}{G}\rho_{eq}+\dot{\rho}_{eq}+3\rho_{eq}\frac{\dot{a}}{a}=-3\frac{P_{\exp}}{c^2}\frac{\dot{a}}{a}. \quad (66)$$

From eq. (13) we conclude that $\frac{\dot{G}}{G}=-\frac{\dot{a}}{a}$. Finally the pressure is obtained from

$$\frac{P_{\exp}}{c^2}=-\frac{2\rho_{m\gamma}}{3}\left(1+B+BL\right). \quad (67)$$

The Einstein Field Equation is completed in a form which eliminates the need for a cosmological constant Λ

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=8\pi GT_{\mu\nu}. \quad (68)$$

7.3. Field Equation Characteristics

All components of the energy-momentum tensor contain $2\rho_{m\gamma}$. By substituting G from eq. (13) and $\rho_{m\gamma}$ from eq. (14) we arrive at the following expression for the tensor

$$T_{\mu\nu} = 3\left(\frac{c}{r_0}\right)^2 \begin{pmatrix} \frac{1+BL}{a^2} & 0 & 0 & 0 \\ 0 & -\frac{1}{3}\cdot\frac{1+B+BL}{a^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{3}\cdot\frac{1+B+BL}{a^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{3}\cdot\frac{1+B+BL}{a^2} \end{pmatrix}. \quad (69)$$

Interestingly G vanishes, the energy-momentum tensor is independent of the gravitational parameter. Making use of the definition of the Hubble parameter in eq. (20) we can write the tensor as follows

$$T_{\mu\nu} = \begin{pmatrix} 3\left(h_H^2 + \frac{c^2}{a^2 r_0^2}\right) & 0 & 0 & 0 \\ 0 & -\left(h_H^2 + (1+B)\frac{c^2}{a^2 r_0^2}\right) & 0 & 0 \\ 0 & 0 & -\left(h_H^2 + (1+B)\frac{c^2}{a^2 r_0^2}\right) & 0 \\ 0 & 0 & 0 & -\left(h_H^2 + (1+B)\frac{c^2}{a^2 r_0^2}\right) \end{pmatrix}. \quad (70)$$

The term 1+B (= β in previous papers, [6]…[9]) indicates the total pressure needed for both expansion and acceleration.

The Field Equation written in its most compact form is now

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}. \quad (71)$$

The Friedmann-Robertson-Walker temporal equation, which also may be called the Hubble expansion equation, is

$$R_{00} - \frac{1}{2}Rg_{00} = T_{00}. \quad (72a)$$

We have

$$-3\frac{\ddot{a}}{a} + 3\frac{\ddot{a}}{a} + 3\left(\frac{\dot{a}}{a}\right)^2 + 3k\frac{c^2}{a^2 r_{cur}^2} = 3h_H^2 + 3\frac{c^2}{a^2 r_0^2}. \quad (72b)$$

Further,

$$\left(\frac{\dot{a}}{a}\right)^2 + k\frac{c^2}{a^2 r_{cur}^2} \equiv h_H^2 + \frac{c^2}{a^2 r_0^2}. \quad (72c)$$

By definition $\frac{\dot{a}}{a} = h_H$. k = 1 means that the universe is closed. $ar_{cur} = ar_0$ indicates that the average curvature radius equals the radius of the observable universe, a circumstance emphasized by Einstein in the cosmological constant paper, [13].

The *acceleration equation* is obtained from

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij}, \quad (73a)$$

where i and j = 1, 2, 3. By substituting elements from eqs (54), (55) and (70) we have

$$\left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2k\frac{c^2}{a^2 r_{cur}^2}\right) - 3\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{c^2}{a^2 r_0^2}\right) \quad (73b)$$
$$= -2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 - \frac{c^2}{a^2 r_0^2} \equiv -h_H^2 - B\frac{c^2}{a^2 r_0^2} - \frac{c^2}{a^2 r_0^2},$$

from which we end up with

$$\ddot{a} = \frac{c^2}{2ar_0^2}B. \quad (74a)$$

or

$$g = r_0\ddot{a} = \frac{c^2}{2ar_0}B. \quad (74b)$$

For a = 1, B = 0,099152544 we have $\mathbf{g_0 = 1{,}0595 \cdot 10^{-11}\ m/s^2}$.

The equation corresponds to our Ansatz in eq. (16). The acceleration g is an inherent characteristic of the expanding universe. Over a comoving distance d in the vicinity of an observer, the scale factor a in eq. (74b) equals 1- $d/r_0 \approx 1$.

As a result, g provides a Coriolis effect, which explains the rotational behaviour of the galaxies and the celestial movements at large, and thereby eliminates the need for dark matter, cf. Eriksson [6].

7.4. Energy Conservation

It is important to prove that the $T_{\mu\nu}$ tensor fulfils the temporal energy conservation condition. The general expression using Christoffel symbols Γ is, cf. Carroll [17],

$$\nabla_\mu T^\mu{}_0 = \partial_\mu T^\mu{}_0 + \Gamma^\mu_{\mu 0}T^0{}_0 - \Gamma^\lambda_{\mu 0}T^\mu{}_\lambda$$
$$= -\partial_0\rho_{eq} - 3\frac{\dot{a}}{a}\left(\rho_{eq} + \frac{P_{exp}}{c^2}\right) = 0. \quad (75a)$$

The time derivative is

$$\partial_0 \rho_{eq} = \frac{\partial}{\partial t}\left(\frac{1+BL}{a^2}\right) = -\left(\frac{2(1+BL)}{a^2} - \frac{B}{a^2}\right)\left(\frac{\dot{a}}{a}\right). \quad (75b)$$

We have

$$\frac{1}{a^2}(2+2BL-B-3-3BL+1+B+BL)\cdot\left(\frac{\dot{a}}{a}\right) = 0. \quad (75c)$$

The conservation condition is fulfilled. The result also indicates that the components ρ_{eq} and P_{exp}/c^2 are correctly derived.

8. The Age of the Universe

The passage of time since the initial event is obtained by integrating the Hubble parameter, eq. (20). The solution, cf. ref. [6], involves the Dawson integral function $D_+(\sqrt{L})$, which can be found in Wolfram Alpha. The cosmic age is

$$t = D_+ \frac{2r}{c\sqrt{B}}. \quad (76)$$

At r = r_0 we have $\sqrt{L}$ = 9,8346789 and further D_+ = 0,0511075. The present age is $\mathbf{t_0 = 4{,}553659 \cdot 10^{17}}$ **s = $\mathbf{14{,}43 \cdot 10^9}$ yr**, slightly larger than the standard value.

9. The Justification of the CBU Theory

The CBU theory is based on ideas presented by prominent physicists, such as Arthur Eddington, Paul Dirac and Fred Hoyle. The theory does not require unrecognized phenomena. In Table 1 some key numbers are compared to recent satellite data, [18].

Considering that CBU is a theoretical construction, wherein the only calibration parameter is the present value of G, the numbers in Table 1 are surprisingly concordant, closest are the Hubble parameter values, difference 1 %.

Table 1. Comparison between the CBU theoretical numbers and recent (2018) data, [18]

	CBU	Satellite
Radius, obs. universe, 10^{26} m	4,20551	4,3963
Universe age, 10^9 yr	14,43	13,79
Hubble parameter H_0, km/sMpc	68,118	67,66
Total real energy W_u, 10^{71} J	1,01795	1,0813
Density of matter and radiation,	0,4544	0,423
Density of dark energy, 10^{-27} kg/m^3	6,113*	5,877
Critical density, 10^{-27}kg/m^3	8,715	8,571

* According to the CBU theory the dark energy is the accumulated value of a virtual ground state energy fraction, from which the real energy emerges.

The largest difference occurs in the density of matter and radiation, difference 7,4 %, partly explained by the difference in r. For satellite data the ratio of the dark and real energy is 13,89, which is fairly close to $\ell_{Pi}/2r_i$. The critical density values are almost identical considering the inaccuracy of observational values. The critical density also proves correctly the assumption that the 3D image (Figure 1) of the universe is spherical, meaning that the total volume is 8 times that of the observable universe.

Figure 2 shows how all principal pieces of a jigsaw puzzle fall into place. The last piece, the Postulate can now be dropped down and complete the picture of the Einsteinian General Relativity and its theoretical background.

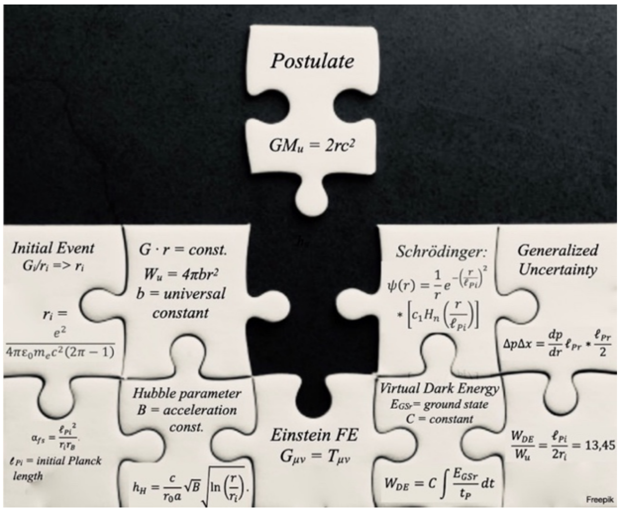

Figure 2. The jigsaw puzzle illustrates how key findings form a comprehensive pattern in support of the CBU theory

10. Conclusions

A detailed investigation and analysis show that the expanding universe according to present observations can be described by known laws of physics and a consistent mathematical framework including the original geodesic formulation of Albert Einstein.

Vacuum energy has been proposed as the source of dark energy, but controversial interpretations of the magnitude of the vacuum energy have not been able to solve the so called cosmological constant problem. In the present study an expression of the quantum ground state energy E_{GSr} was derived as a function of the curvature radius r. E_{GSr} is considered as the energy of a foam of virtual particles, from which real energy emerges as positron-electron pairs, a process controlled by the generalized uncertainty principle. The approach leads to a very plausible solution of the dark energy problem.

The Cosmic Microwave Background (CMB) requires a comment. An embryo to a theory suggests that the initiatory homogeneous black hole undergo a transition, whereby the radiation due to positron-electron annihilations is released and the rest of matter form a multitude (10^{12}) of black hole "droplets". The reason for a transition might be that the energy reaches a stage of half matter and half radiation, thus satisfying the Schwarzschild condition $r_u = 2GM/c^2$. This scenario suits the CMB homogeneity pattern and also explains why the CMB image roughly correlates with the present distribution of galaxies.

According to the CBU theory matter is created spontaneously throughout space. So far, the change of electrons and positrons into fermions is an open question. Do the black holes in the galaxy centres have an impact?

Does the gravitational pull near the black holes make positron-electron clouds implode and create the nucleus of new stars? A change of paradigm could bring new answers to these questions.

The CBU theory opens new avenues for the understanding of the distinction between our perception of reality and the quantum world.

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